

Warm Inflation: Towards a realistic COBE data power spectrum for matter and metric thermal coupled fluctuations

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I consider the COBE data coarse - grained field that characterize the now observable universe for a model of warm inflation which takes into account the thermal coupled fluctuations of the scalar field with the thermal bath. The power spectrum for both, matter and metric fluctuations are analyzed. I find that the amplitude for the fluctuations of the metric when the horizon entry, should be very small for the expected values of temperature.

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I. INTRODUCTION

Warm inflation takes into account separately, the matter and radiation energy fluctuations. In this scenario the matter field φ interacts with the particles of a thermal bath with mean temperature T_r , which is smaller than the Grand Unified Theories (GUT) critical temperature $T_r < T_{GUT} \simeq 10^{15}$ GeV. This scenario was firstly studied by Berera [1]. The warm inflation scenario served as a explicit demonstration that inflation can occur in the presence of a thermal component. In the formalism developed by Berera the temperature of the universe remains constant during the inflationary expansion.

Warm inflation was originally formulated in a phenomenological setting, but some attempts of a fundamental justification has also been presented [2]. Furthermore, a dynamical system analysis showed that a smooth transition from inflationary to a radiation phase is attained for many values of the friction parameter, thereby showing that the warm inflation scenario may be a workable variant to standard inflation. During the warm inflationary era, vacuum fluctuations on scales smaller than the size of the horizon are magnified into classical perturbations on scales bigger than the Hubble radius. The classical perturbations can lead to an effective curvature of spacetime and energy density perturbations [3].

In an alternative formalism for warm inflation, I studied a model where the mean temperature and the amplitude of the temperature's fluctuations decreases with time for a rapid power - law expanding universe. This is the most significant difference with the Berera's formalism in which the warm inflation expansion is isothermal [4,5,6]. During the warm inflationary expansion, the kinetic energy density ρ_{kin} is smaller with respect to the vacuum energy

$$\rho(\varphi) \sim \rho_m \sim V(\varphi) \gg \rho_{kin}.$$

The kinetic energy density is given by

$$\rho_{kin} = \rho_r(\varphi) + \frac{\dot{\varphi}^2}{2},$$

where

$$\rho_r(\varphi) = \frac{\tau(\varphi)}{8H(\varphi)}\dot{\varphi}^2. \quad (1)$$

Here, the dot denotes the derivative with respect to the time. Furthermore, $\tau(\varphi)$ and $H(\varphi)$ are the friction and Hubble parameters. The eq. (1) comes from the assumption that the radiation energy density remains constant during the inflationary era ($\dot{\rho}_r \simeq 0$).

The Lagrangian that describes the warm inflation scenario is

$$\mathcal{L}(\varphi, \varphi_{,\mu}) = -\sqrt{-g} \left[\frac{R}{16\pi} + \frac{1}{2}g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} + V(\varphi) \right] + \mathcal{L}_{int}, \quad (2)$$

where R is the scalar curvature, $g^{\mu\nu}$ gives the metric tensor and g is the metric. The Lagrangian \mathcal{L}_{int} takes into account that the particles in the thermal bath interact with the scalar field φ . In principle, a permanent or temporary coupling of the scalar field φ with others fields might also lead to dissipative processes producing entropy at different eras of the cosmic evolution. It is expected that progress in nonequilibrium statistics of quantum fields will provide the necessary theoretical framework for discussing dissipation in more general cases [7].

The semiclassical Friedmann equation is

$$H^2(\varphi) = \frac{8\pi}{3M_p^2} \langle E | \rho_m(\varphi) + \rho_r(\varphi) | E \rangle, \quad (3)$$

where $M_p = 1.2 \times 10^{19}$ GeV is the Planckian mass.

Now I consider the semiclassical expansion for the inflaton field φ

$$\varphi(\vec{x}, t) = \phi_c(t) + \alpha(t)\phi(\vec{x}, t). \quad (4)$$

Here, $\phi_c(t) = \langle E | \varphi | E \rangle$, $\langle E | \phi | E \rangle = \langle E | \dot{\phi} | E \rangle = 0$, and $|E\rangle$ is an arbitrary state. Furthermore, $\alpha(t)$ is a dimensionless time - dependent function that characterize the gravitational coupling between the fluctuations of the matter field and the fields in the thermal bath. A lot of work can be done on phenomenological grounds, as, for instance, by applying nonequilibrium thermodynamic techniques to the problem or even studying particular models with dissipation. An example of this latter case is the warm inflationary picture recently proposed [1].

The aim of this work is the study of the power spectrum in warm inflation with the semiclassical expansion (4), taking into account the COBE data coarse - grained field introduced in a previous work [8]. This topic was studied in [8] but with the semiclassical expansion $\varphi = \phi_c + \phi$. In this work I incorporate in the formalism the backreaction of the metric for the study of the effective curvature for the now observable universe, when the fluctuations are coupled with the thermal bath.

II. DYNAMICS OF THE INFLATON

A. Dynamics of the classical field

The dynamics for the classical field in warm inflation was obtained in previous works [4,5,6]. The equation of motion for ϕ_c is

$$\ddot{\phi}_c + [3H_c + \tau_c]\dot{\phi}_c + V'(\phi_c) = 0, \quad (5)$$

where $H_c \equiv H(\phi_c) + \frac{\dot{a}}{a}$, and $\tau_c \equiv \tau(\phi_c)$ and $V'(\phi_c) \equiv \frac{dV(\varphi)}{d\varphi} \Big|_{\phi_c}$. The term $\tau_c\dot{\phi}_c$ in eq. (5) shows as the scalar field evolves with the time in a damped regime generating an expansion which depends on the mean temperature T_r

of the thermal bath. As a consequence, the subsequent reheating mechanism is not needed and thermal fluctuations produce the primordial spectrum of density perturbations [5,6]. Furthermore, $\dot{\phi}_c = -\frac{M_p^2}{4\pi} H'_c \left(1 + \frac{\tau_c}{3H_c}\right)^{-1}$ and the classical effective potential is

$$V(\phi_c) = \frac{M_p^2}{8\pi} \left[H_c^2 - \frac{M_p^2}{12\pi} (H'_c)^2 \left(1 + \frac{\tau_c}{4H_c}\right) \times \left(1 + \frac{\tau_c}{3H_c}\right)^{-2} \right]. \quad (6)$$

The radiation energy density of the background is

$$\rho_r[\phi_c(t)] \simeq \frac{\tau_c}{8H_c} (H'_c)^2 \left(\frac{M_p^2}{4\pi}\right)^2 \left(1 + \frac{\tau_c}{3H_c}\right)^{-2}, \quad (7)$$

and the temperature of this background is

$$T_r \propto \rho^{1/4}[\phi_c(t)]. \quad (8)$$

Note that the temperature depends on time. In the warm inflation model here studied, I will suppose that it decreases with time, in agreement one expects in an expanding universe. The temporal evolution of the background temperature depends on the particular model which one considers. For example, in a power-law expanding universe $T_r \sim t^{-1/2}$ [6].

B. First order ϕ - fluctuations

In this section I will study the first order ϕ - fluctuations for the matter field φ , on a globally flat Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = -dt^2 + a^2 d\vec{x}^2, \quad (9)$$

which describes a globally isotropic and homogeneous spacetime. The equation of motion for the quantum perturbations ϕ , is

$$\ddot{\phi} + \left[2\frac{\dot{\alpha}}{\alpha} + (3H_c + \tau_c) \right] \dot{\phi} - \frac{1}{a^2} \nabla^2 \phi + \left[(3H_c + \tau_c) \frac{\dot{\alpha}}{\alpha} + V''(\phi_c) \right] \phi = 0. \quad (10)$$

The function $\alpha(t)$ depends on time. I consider $\alpha(t) = F[T_r(t)/M]$, where $T_r(t)$ is the temperature of the background and $M \simeq 10^{15}$ GeV is the GUT mass. The structure of the equation (10) can be simplified by means of the map $\chi = e^{3/2 \int (H_c + \tau_c/3 + \frac{2\dot{\alpha}}{3\alpha}) dt} \phi$

$$\ddot{\chi} - \frac{1}{a^2} \nabla^2 \chi - \mu^2(t) \chi = 0, \quad (11)$$

where $\mu^2(t) = \frac{k_o^2}{a^2}$ is the time dependent parameter of mass and $k_o(t)$ is the time dependent wave number which

separates the long wavelength ($k \ll k_o$) and the short wave ($k \gg k_o$) sectors.

The square time dependent parameter of mass is

$$\mu^2(t) = \frac{9}{4} \left(H_c + \frac{\tau_c}{3} \right)^2 - V''(\phi_c) + \frac{3}{2} \left(\dot{H}_c + \frac{\dot{\tau}_c}{3} \right). \quad (12)$$

Note that $\mu(t)$ do not depends on the function $\alpha(t)$.

C. Second order ϕ - fluctuations and Backreaction

Making a second order ϕ - fluctuations expansion for φ , one obtains the following semiclassical Friedmann equation

$$H_c^2 + \frac{K}{a^2} = \frac{8\pi}{3M_p^2} \langle E | \rho_m + \rho_r | E \rangle, \quad (13)$$

where K is an effective curvature produced by the backreaction of the metric with the fluctuations of the scalar field. This curvature is given by

$$\frac{K}{a^2} = \frac{8\pi}{3M_p^2} \left[\left(1 + \frac{\tau_c}{8H_c} \right) \left(\frac{\dot{\alpha}^2}{2} \langle \phi^2 \rangle + \frac{\alpha^2}{2} \langle \dot{\phi}^2 \rangle + \alpha \dot{\alpha} \langle \phi \dot{\phi} \rangle \right) + \frac{\alpha^2}{a^2} \left\langle \left(\vec{\nabla} \phi \right)^2 \right\rangle + \frac{V''}{2} \alpha^2 \langle \phi^2 \rangle \right]. \quad (14)$$

Note that K depends on the temporal evolution of $\alpha(t)$ as well as the expectation values for ϕ^2 , $\phi \dot{\phi}$, $\dot{\phi}^2$ and $\left(\vec{\nabla} \phi \right)^2$. If $\alpha(t)$ is a function of the temperature, $\alpha(t) = F[T_r(t)/M]$, the instantaneous comoving temperature will be very important during the warm inflationary regime.

To study the backreaction of the metric with the fluctuations ϕ , I introduce the metric

$$ds^2 = -dt^2 + a^2 [1 + h(\vec{x}, t)] d\vec{x}^2, \quad (15)$$

where $h(\vec{x}, t)$ represents the fluctuations of the metric produced by the ϕ - fluctuations. Making the following expansion for $H(\varphi)$

$$H[\varphi(\vec{x}, t)] \simeq H_c[\phi_c(t)] + H'[\phi_c(t)] \alpha(t) \phi(\vec{x}, t), \quad (16)$$

one obtains the following expression for $h(\vec{x}, t)$ [3]

$$h(\vec{x}, t) \simeq 2 \int^t dt' \alpha(t') \phi(\vec{x}, t') H'[\phi_c(t')], \quad (17)$$

and the effective curvature can be represented by

$$\begin{aligned} \frac{K}{a^2} &= \left\langle E \left| \left(\frac{\dot{h}(\vec{x}, t)}{2} \right)^2 \right| E \right\rangle \\ &= \left\langle E \left| [\alpha(t) \phi(\vec{x}, t) H'(\phi_c)]^2 \right| E \right\rangle. \end{aligned} \quad (18)$$

This expression shows that the temporal evolution of the effective curvature arises from the matter field fluctuations $\phi(\vec{x}, t)$ and the temperature of the thermal bath, due to the fact I am considering that $\alpha(t)$ is a function of the temperature of this bath. In order to study the evolution of the fluctuations on the infrared (long wavelength) sector, firstly one can write the fields χ and h as two Fourier expanded fields

$$\chi(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left[a_k \chi_k(\vec{x}, t) + a_k^\dagger \chi_k^*(\vec{x}, t) \right], \quad (19)$$

$$h(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left[a_k h_k(\vec{x}, t) + a_k^\dagger h_k^*(\vec{x}, t) \right], \quad (20)$$

where $\chi_k(\vec{x}, t) = e^{i\vec{k} \cdot \vec{x}} \xi_k(t)$ and $h_k(\vec{x}, t) = e^{i\vec{k} \cdot \vec{x}} \tilde{\xi}_k(t)$. Here, $\tilde{\xi}_k(t) = 2 \int^t dt' \alpha(t') H'[\phi_c(t')] \xi_k(t')$ and the asterisk denote the complex conjugate. The operators a_k and a_k^\dagger are the well known annihilation and creation operators, which satisfy $[a_k, a_{k'}^\dagger] = i\delta^{(3)}(k - k')$ and $[a_k^\dagger, a_{k'}^\dagger] = [a_k, a_{k'}] = 0$. The commutation relations for the fields χ and h are

$$[\chi(\vec{x}, t), \dot{\chi}(\vec{x}', t)] = \frac{1}{(2\pi)^3} \int d^3k \left(\xi_k \dot{\xi}_k^* - \dot{\xi}_k \xi_k^* \right) e^{-i\vec{k} \cdot (\vec{x} - \vec{x}')}, \quad (21)$$

$$[h(\vec{x}, t), \dot{h}(\vec{x}', t)] = \frac{1}{(2\pi)^3} \int d^3k \left(\tilde{\xi}_k \dot{\tilde{\xi}}_k^* - \dot{\tilde{\xi}}_k \tilde{\xi}_k^* \right) e^{-i\vec{k} \cdot (\vec{x} - \vec{x}')}. \quad (22)$$

To obtain $[\chi(\vec{x}, t), \dot{\chi}(\vec{x}', t)] = i\delta^{(3)}(\vec{x} - \vec{x}')$ in eq. (21), one requires that $(\xi_k \dot{\xi}_k^* - \dot{\xi}_k \xi_k^*) = i$.

III. DATA COBE COARSE - GRAINED FIELDS AND STOCHASTIC REPRESENTATION

The data COBE coarse - grained matter field χ_{Ccg} were introduced in a previous work [5]

$$\chi_{Ccg} = \frac{1}{(2\pi)^{3/2}} \int d^3k G(k, t) \left[a_k \chi_k + a_k^\dagger \chi_k^* \right]. \quad (23)$$

Now we can introduce the data COBE coarse - grained field h_{Ccg} for the fluctuations of the metric

$$h_{Ccg} = \frac{1}{(2\pi)^{3/2}} \int d^3k G(k, t) \left[a_k h_k + a_k^\dagger h_k^* \right]. \quad (24)$$

In eqs. (23) and (24) the suppression factor $G(k, t)$ is given by [8]

$$G(k, t) = \sqrt{\frac{1}{1 + \left(\frac{k_o(t)}{k} \right)^N}}, \quad (25)$$

with $N = m - n$. Causality places a strict constraint on the suppression index: $N \geq 4 - n$. A suppression factor

like (25) also was founded in a model with cosmic strings plus cold or hot dark matter [9] Furthermore, the square fluctuations for the data COBE coarse - grained matter field is

$$\begin{aligned} \langle E | \chi_{Ccg}^2 | E \rangle &= \int_0^\infty \frac{dk}{k} \mathcal{P}_{\chi_{Ccg}}(k) \\ &= \frac{1}{2\pi^2} \int_0^{k_o(t)} dk k^2 |\xi_k(t)|^2 G^2(k, t), \end{aligned} \quad (26)$$

where the power spectrum $\mathcal{P}_{\chi_{Ccg}}(k)$ when the horizon exit is [10]

$$\mathcal{P}_{\chi_{Ccg}}(k) = A(t_*) \left(\frac{k}{k_o(t_*)} \right)^n f(k). \quad (27)$$

Here, t_* denotes the time when the horizon entry, for which $k_o(t_*) \simeq \pi H_o$ in comoving scale. The parameters in eq. (27) are the amplitude $A(t_*)$ on time t_* , the spectral index n , the suppression wavenumber k_o and the suppression index m .

The stochastic equation for χ_{Ccg} is [8]

$$\ddot{\chi}_{Ccg} - \left(\frac{k_o(t)}{a(t)} \right)^2 \chi_{Ccg} = \frac{N}{k_o(t)} [\xi_1 + \xi_2], \quad (28)$$

where

$$\begin{aligned} \xi_1(\vec{x}, t) &= -\frac{\dot{k}_o k_o^N}{(2\pi)^{3/2}} \int d^3k k^{-N} G^3(k, t) \\ &\quad \times \left[a_k \dot{\chi}_k + a_k^\dagger \dot{\chi}_k^* \right], \end{aligned} \quad (29)$$

$$\begin{aligned} \xi_2(\vec{x}, t) &= -\frac{\dot{k}_o k_o^N}{(2\pi)^{3/2}} \int d^3k k^{-N} G^5(k, t) \\ &\quad \times \left[\left(\frac{k_o}{k} \right)^N \left(3\dot{k}_o^2 - 2k_o \ddot{k}_o \right) + 2\dot{k}_o^2(1 - N) - 2k_o \ddot{k}_o \right] \\ &\quad \times \left[a_k \chi_k + a_k^\dagger \chi_k^* \right]. \end{aligned} \quad (30)$$

The noises (29) and (30) arise from the increasing number of degrees of freedom of the infrared sector from the short-wavelength sector. For the special case considered in eq. (25), ξ_1 is a colored noise, while ξ_2 gives non - local dissipation.

Since $h \simeq 2 \int^t dt' \alpha(t') H'(t') \phi(\vec{x}, t)$, one can rewrite it as $h \simeq 2 \int^t dt' \tilde{\alpha}(t') H'(t') \chi(\vec{x}, t)$, where $\tilde{\alpha}(t) = e^{-3/2 \int dt (H_c + \tau_c/3 + \frac{2\tilde{\alpha}}{3\alpha})} \alpha(t)$. With this representation for h , the data COBE coarse - grained metric field h_{Ccg} becomes

$$h_{Ccg}(\vec{x}, t) \simeq 2 \int^t dt' \tilde{\alpha}(t') H'(t') \chi_{Ccg}(\vec{x}, t'). \quad (31)$$

Replacing (31) in eq. (28), one obtains the following stochastic equation for h_{Ccg}

$$\frac{\partial^3 h_{Ccg}}{\partial t^3} + 2 \frac{\dot{g}(t)}{g(t)} \frac{\partial^2 h_{Ccg}}{\partial t^2} - \left(\frac{k_o}{a} \right)^2 \frac{\partial h_{Ccg}}{\partial t} + \frac{\ddot{g}(t)}{g(t)} h_{Ccg} = \frac{N}{g(t) k_o(t)} [\xi_1(\vec{x}, t) + \xi_2(\vec{x}, t)], \quad (32)$$

where $g(t) = [2\tilde{\alpha}(t)H'(t)]^{-1}$. The square fluctuations for the field ϕ_{Ccg} is

$$\langle E | \phi_{Ccg}^2 | E \rangle = \frac{e^{-3 \int (H_c + \frac{\tau_c}{3} + \frac{2\dot{\alpha}}{3\alpha}) dt}}{2\pi^2} \times \int_0^{k_o} dk k^2 |\xi_k^2(t)| G^2(k, t). \quad (33)$$

Thus, the effective curvature K/a^2 for the now observable universe is [see eq. (18)]

$$\frac{K}{a^2} \Big|_{\text{COBE}} = [\alpha(t) H'[\phi_c(t)]]^2 \langle E | \phi_{Ccg}^2 | E \rangle. \quad (34)$$

Hence, the power spectrum for ϕ_{Ccg} and h_{Ccg} when the horizon entry, are

$$\mathcal{P}_{\phi_{Ccg}}(k) = B(t_*) \left(\frac{k}{k_o(t_*)} \right)^n f(k), \quad (35)$$

$$\mathcal{P}_{h_{Ccg}}(k) = C(t_*) \left(\frac{k}{k_o(t_*)} \right)^n f(k). \quad (36)$$

Here, $B(t_*)$ and $C(t_*)$ are the amplitude such that

$$B(t_*) = A(t_*) e^{-3 \int^{t_*} (H_c + \frac{\tau_c}{3} + \frac{2\dot{\alpha}}{3\alpha}) dt}, \quad (37)$$

$$C(t_*) = [\alpha(t) H'[\phi_c(t)]]^2 \Big|_{t=t_*} B(t_*). \quad (38)$$

Due to $|\delta_k|^2 = \mathcal{P}_{\phi_{Ccg}}(k)$ [8], the spectral density become $|\delta_k| = k^n f(k)$. The standard choice $n = 1$ and $f(k)$ as constant, was invoked by Harrison [11] and Zel'dovich [12] on the grounds that it is scale invariant at the epoch of the horizon entry. The constraint $|n - 1| < 0.3$ was obtained from the data COBE spectrum [9]. Note that both $B(t_*)$ and $C(t_*)$ depends on the temperature of the background when the horizon entry. This is a very important characteristic that becomes from this formulation, once one consider $\varphi = \phi_c + \alpha(t)\phi$ and $H(\varphi) = H_c + \alpha(t)H'\phi$ as semiclassical expansions for φ and $H(\varphi)$. From eq. (38) one obtains

$$\frac{C(t_*)}{B(t_*)} = [\alpha(t) H'[\phi_c(t)]]^2 \Big|_{t=t_*}. \quad (39)$$

Taking $\rho_r(t_*) = \frac{\pi^2}{30} N[T_r(t_*)] T_r^4(t_*)$, where $N[T_r(t_*)]$ is the number of relativistic degrees of freedom at temperature $T_r(t_*)$ and replacing $(H'_c)^2$ in eq. (7), one obtains (for $N[T_r(t_*)] \simeq 10^3$, $\alpha(t_*) = \left(\frac{T_r(t_*)}{M} \right)^\beta$ — $\beta \geq 0$ — and $M \simeq 10^{-4} M_p$)

$$\frac{C(t_*)}{B(t_*)} = \frac{64}{45} \pi^4 10^{3+8\beta} \frac{(3H_c + \tau_c)^2}{3H_c \tau_c} \left(\frac{T_r(t_*)}{M_p} \right)^{2(\beta+2)}. \quad (40)$$

For the case $\tau_c(t_*) \simeq H_c(t_*)$, one obtains the expression

$$\frac{C(t_*)}{B(t_*)} \simeq 10^{(6+8\beta)} \left(\frac{T_r(t_*)}{M_p} \right)^{2(\beta+2)}. \quad (41)$$

For example, for $\beta = 1$ one obtains $T_r(t_*) = 10^{-5} M_p$ for $\frac{C(t_*)}{B(t_*)} \simeq 10^{-15}$. This implies that the amplitude for the fluctuations of the metric when the horizon entry, should be very small for the expected values of temperature.

IV. FINAL REMARKS

To summarize, in this letter I considered a model for warm inflation where the fluctuations of the scalar field are coupled with the thermal bath. This coupling, depends on the temperature of the background which is a function of the temperature. The temperature decreases with the time, as well as the Hubble parameter.

By means of the COBE data coarse - grained field for the fluctuations of the scalar field I characterize these fluctuations on the scale of the now observable universe. Once one knows the stochastic equation for the field χ_{Ccg} , it is possible to obtain the stochastic equation for h_{Ccg} [see eq. (32)]. The square fluctuations for ϕ_{Ccg} and h_{Ccg} give the spectral density δ_k for both, ϕ_{Ccg} and h_{Ccg} . The spectral density δ_k depends on the function $G^2(k, t)$, the modes ξ_k and the index n . Finally, I find that the amplitude for the fluctuations of the metric when the horizon entry, should be very small for the expected values of temperature.

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